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MOTION OF A VERTICAL WALL FIXED ON SPRINGS UNDER THE ACTION OF SURFACE WAVES

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A two-dimensional unsteady hydroelastic problem of interaction between surface waves and a moving vertical wall fixed on springs is studied. An analytical solution of the problem is constructed using a linear approximation, and a numerical solution within the framework of a nonlinear model of a potential fluid flow is found by a complex boundary element method. By means of analysis of the linear and nonlinear solutions, it is found that the linear solution can be used to predict some important characteristics of the wall motion and the fluid flow in the case of moderate wave amplitudes.

Key words: *hydroelasticity, unsteady action, moving vertical wall, nonlinear surface waves, complex boundary element method.*

The behavior of elastic walls of large structures subject to surface waves is one of the main problems of hydroelasticity. Deformations of the walls are caused by the wave motion of the fluid, which, in turn, depends on elastic bending of the walls. The unsteadiness and nonlinearity of the processes should be taken into account.

Here we consider only two-dimensional problems where the fluid occupies a semi-bounded channel of constant depth with an elastic vertical wall. Some publications described vibrations of the side wall modeled by an elastic beam under the action of incoming periodic waves (see, e.g., [1] and the references therein). A linearized problem was considered, and the motions of the fluid and the beam were assumed to be periodic functions of time.

Unsteady linear wave motions of the fluid initiated by initial deformation of the elastic side wall were considered in [2].

A simpler formulation can be obtained in the problem of interaction of surface waves with a solid vertical wall fixed on springs to a motionless unit. This problem was chosen to test numerical algorithms developed for solving nonlinear problems of hydroelasticity in the case of significant amplitudes of the surface waves interacting with the structure. It is assumed that the wall motion is frictionless and there is no penetration of the fluid between the wall and the channel bottom. Such a structure can be used as an element of the wave power station for accumulating the energy of surface waves. The results of the numerical solution of this problem in the nonlinear formulation by the finite-difference method were briefly described in [3]. The surface waves were generated by initial perturbation of the free surface. The free surface dynamics was compared for the cases of the moving and motionless side walls. The calculations described in [3], however, were performed without accounting for the contribution of the initial elevation of the free surface to the distribution of the hydrostatic load acting on the vertical wall. In the present work, the problem is solved with this initial elevation taken into account, and the solutions of the linear and nonlinear problems are compared. The linear problem is solved both by the method of integral transformations and numerically, by the complex boundary element method (CBEM). This method is also used to solve the nonlinear problem.

1. Formulation of the Problem. We consider a plane unsteady problem of the wave motion of an ideal incompressible fluid in the domain with a moving wall (Fig. 1). In the undisturbed state, the fluid layer of depth

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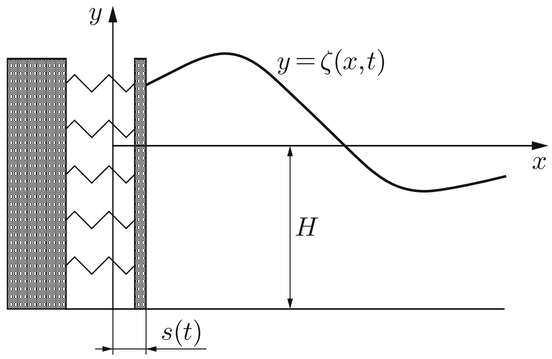

Fig. 1. Sketch of the flow.

H is bounded by the free surface from above and by a solid horizontal bottom from below. The semi-infinite layer of the fluid is bounded on the left by a wall, which remains in the vertical position during the entire period of its motion. The wall displacements are determined the hydrodynamic loads and by the restoring force which is assumed to be proportional to the wall displacements. The Cartesian coordinate system is arranged so that the x axis coincides with the undisturbed free surface, and the y axis is directed vertically upward and corresponds to the wall position at the initial time. The wall displacement is denoted by $s(t)$, with $s(0) = 0$ and $\dot{s}(0) = 0$. The dot indicates differentiation with respect to time t. The equation

$$y = \zeta(x,t) \qquad [x \geqslant s(t)]$$

describes the elevation of the free surface of the fluid during its motion. The shape of the free surface at the initial time $t = 0$ is described by the equation

$$y = \zeta(x,0) \equiv \zeta_0(x) \qquad (x \geqslant 0), \tag{1.1}$$

where the function $\zeta_0(x)$ is given [$\zeta_0(0) = 0$ and $\zeta_0(x) \to 0$ as $x \to \infty$]. At $t > 0$, the fluid flow is caused by the initial elevation of the free boundary (1.1) and by the associated wall motion; it is a potential flow. In the flow region which varies with time,

$$\Omega(t) = \{x, y\colon \quad x > s(t), \quad -H < y < \zeta(x,t)\}, \tag{1.2}$$

the velocity potential $\varphi(x,y,t)$ satisfies the Laplace equation

$$\Delta\varphi = 0 \qquad [x, y \in \Omega(t)] \tag{1.3}$$

with the boundary conditions

$$\frac{\partial\varphi}{\partial x} = \dot{s}(t) \qquad (x = s(t), \quad -H \leqslant y \leqslant \zeta_*(t)); \tag{1.4}$$

$$\frac{\partial\varphi}{\partial y} = 0 \qquad (x \geqslant s(t), \quad y = -H); \tag{1.5}$$

$$\frac{\partial\varphi}{\partial y} = \frac{\partial\zeta}{\partial t} + \frac{\partial\zeta}{\partial x}\frac{\partial\varphi}{\partial x}, \qquad p = 0 \qquad (x \geqslant s(t), \quad y = \zeta(x,t)); \tag{1.6}$$

$$\varphi \to 0 \qquad (x \to +\infty), \tag{1.7}$$

where $\zeta_*(t) = \zeta[s(t), t]$ is the elevation of the fluid on the moving wall.

The hydrodynamic pressure $p(x,y,t)$ is calculated by the Cauchy–Lagrange integral as

$$p = -\rho\Big(\frac{\partial\varphi}{\partial t} + \frac{1}{2}\,|\nabla\varphi|^2 + gy\Big) \tag{1.8}$$

(ρ is the fluid density and g is the acceleration due to gravity). At the initial time, we have

$$\varphi(x,y,0) = 0, \qquad \frac{\partial\varphi}{\partial t}(x, \zeta_0(x), 0) = -g\zeta_0(x). \tag{1.9}$$

At $t = 0$, the pressure distribution along the wall is calculated by the formula

$$p(0, y, 0) = -\rho\Big(\frac{\partial\varphi}{\partial t}\Big|_{\substack{x=0\\ t=0}} + gy\Big),$$

and its value is equal to zero on the free surface of the fluid. The force $F(0)$ acting on the vertical wall at the initial time is balanced by the restoring force of the springs. The equation of the horizontal displacement of the plate has the form

$$m\ddot{s} + \gamma s = F(0) - F(t) \qquad (t > 0), \tag{1.10}$$

where

$$F(t) = \int_{-H}^{\zeta_*(t)} p[s(t), y, t]\, dy, \tag{1.11}$$

m is the plate mass, and γ is the stiffness coefficient of the springs.

2. Linear Approximation. The linear approximation can be used to determine the wall motion and the wave motion of the fluid in the case of small initial perturbations of the free surface. In the linear wave theory, the boundary conditions (1.6) are simplified:

$$\frac{\partial\varphi}{\partial y} = \frac{\partial\zeta}{\partial t}, \qquad \frac{\partial\varphi}{\partial t} + g\zeta = 0 \qquad (x \geqslant 0, \quad y = 0). \tag{2.1}$$

On the vertical wall, condition (1.4) is replaced by the linearized boundary condition

$$\frac{\partial\varphi}{\partial x} = \dot{s}(t) \qquad (x = 0, \quad -H \leqslant y \leqslant 0), \tag{2.2}$$

which allows us to consider the problem in a fixed domain which does not vary in time:

$$\Omega_1 = \{x, y\colon \quad x \geqslant 0, \quad -H \leqslant y \leqslant 0\}.$$

The hydrodynamic pressure is determined from the linearized equation (1.8):

$$p = -\rho\Big(\frac{\partial\varphi}{\partial t} + gy\Big),$$

and the force acting on the vertical wall is

$$F(t) = \int_{-H}^{0} p(0, y, t)\, dy. \tag{2.3}$$

For the velocity potential $\varphi(x, y, t)$, the solution of the Laplace equation (1.3) in the domain Ω_1 with the boundary conditions (1.5), (1.7), (2.1), and (2.2) and the initial conditions

$$\zeta(x, 0) = \zeta_0(x), \qquad \varphi(x, y, 0) = 0,$$

$$\frac{\partial\varphi}{\partial t} = -g\zeta_0(x) \qquad (x \geqslant 0, \quad y = 0, \quad t = 0)$$

is sought in the form of a sum of three unknown functions:

$$\varphi(x, y, t) = \varphi_1(x, y, t) + \varphi_2(x, y, t) + \varphi_3(x, y, t).$$

Correspondingly, the elevation of the free surface $\zeta(x, t)$ can be presented as a sum of three terms:

$$\zeta(x, t) = \zeta_1(x, t) + \zeta_2(x, t) + \zeta_3(x, t).$$

The functions $\varphi_j(x, y, t)$ $(j = 1, 2, 3)$ satisfy the Laplace equation

$$\Delta\varphi_j = 0 \qquad (x, y \in \Omega_1)$$

with the boundary condition

$$\frac{\partial\varphi_j}{\partial y} = 0 \qquad (x \geqslant 0, \quad y = -H)$$

and decrease at infinity as $x \to +\infty$.

The function $\varphi_1(x, y, t)$ is the solution of the Cauchy–Poisson problem for a motionless wall with the following boundary and initial conditions:

$$\frac{\partial \varphi_1}{\partial t} + g\zeta_1 = 0, \qquad \frac{\partial \varphi_1}{\partial y} = \frac{\partial \zeta_1}{\partial t} \qquad (x \geqslant 0, \quad y = 0),$$

$$\frac{\partial \varphi_1}{\partial x} = 0 \qquad (x = 0, \quad -H < y < 0),$$

$$\zeta_1(x, 0) = \zeta_0(x), \qquad \varphi_1(x, y, 0) = 0.$$

The velocity potential $\varphi_2(x, y, t)$ describes the flow of an incompressible fluid initiated by the motion of the vertical wall without accounting for gravity. The boundary conditions for this function have the form

$$\varphi_2 = 0 \qquad (x \geqslant 0, \quad y = 0),$$
$$\frac{\partial \varphi_2}{\partial x} = \dot{s}(t) \qquad (x = 0, \quad -H \leqslant y \leqslant 0). \tag{2.4}$$

The function $\varphi_3(x, y, t)$ satisfies the following boundary and initial conditions:

$$\frac{\partial \varphi_3}{\partial t} + g\zeta_3 = -g\zeta_2, \qquad \frac{\partial \varphi_3}{\partial y} = \frac{\partial \zeta_3}{\partial t} \qquad (x \geqslant 0, \quad y = 0),$$

$$\frac{\partial \varphi_3}{\partial x} = 0 \qquad (x = 0, \quad -H \leqslant y \leqslant 0),$$

$$\varphi_3(x, y, 0) = 0, \qquad \zeta_3(x, 0) = 0.$$

The velocity potential $\varphi_1(x, y, t)$ and the free boundary elevation $\zeta_1(x, y, t)$ are obtained using an integral Fourier transform with respect to x and a Laplace transform with respect to t in the form

$$\varphi_1(x, y, t) = -\frac{2g}{\pi} \int_0^\infty \frac{\cosh\,[k(y + H)] \cos\,(kx)}{\omega(k) \cosh\,(kH)}\, \tilde{\zeta}_0(k) \sin\,[\omega(k)t]\, dk,$$
$$\zeta_1(x, t) = \frac{2}{\pi} \int_0^\infty \cos\,(kx)\, \tilde{\zeta}_0(k) \cos\,[\omega(k)t]\, dk, \tag{2.5}$$

where

$$\tilde{\zeta}_0(k) = \int_0^\infty \zeta_0(x) \cos\,(kx)\, dx, \qquad \omega(k) = \sqrt{gk \tanh\,(kH)}.$$

The potential $\varphi_2(x, y, t)$ can be found by two methods. The first method described in [4] can be applied to both finite-depth and infinite-depth fluids. In accordance with this method, the solution $\varphi_2(x, y, t)$ has the form

$$\varphi_2(x, y, t) = -\frac{\dot{s}(t)}{2\pi} \int_{-H}^0 \Big(\ln \frac{\cosh\,(bx) + \cos\,[b(\beta + y + 2H)]}{\cosh\,(bx) - \cos\,[b(\beta + y + 2H)]} + \ln \frac{\cosh\,(bx) + \cos\,[b(\beta - y)]}{\cosh\,(bx) - \cos\,[b(\beta - y)]}\Big)\, d\beta,$$

$$b = \pi/(2H).$$

The second method is simpler, but it can be used only for finite-depth fluids. The solution for $\varphi_2(x, y, t)$ is sought in the form

$$\varphi_2(x, y, t) = \dot{s}(t) \sum_{n=0}^\infty a_n \exp\,(-\lambda_n x) \cos\,[\lambda_n(y + H)], \qquad \lambda_n = \frac{\pi(2n + 1)}{2H}, \tag{2.6}$$

where the coefficients a_n are unknown. Under the boundary condition (2.4), we have

$$a_n = (-1)^{n+1} \frac{8H}{\pi^2(2n + 1)^2}.$$

In view of Eq. (2.6), the solutions for $\varphi_2(x, y, t)$ and $\zeta_2(x, t)$ are written as

$$\varphi_2(x, y, t) = \frac{8\dot{s}(t)H}{\pi^2} \sum_{n=0}^{\infty} (-1)^{n+1} \frac{\exp(-\lambda_n x) \cos[\lambda_n(y + H)]}{(2n + 1)^2},$$

$$\zeta_2(x, t) = \frac{4s(t)}{\pi} \sum_{n=0}^{\infty} \frac{\exp(-\lambda_n x)}{2n + 1} = \frac{4s(t)}{\pi} \operatorname{arctanh}\Big[\exp\Big(-\frac{\pi x}{2H}\Big)\Big].$$

Using the solutions obtained and the integral Fourier and Laplace transforms, we can write the solutions for $\varphi_3(x, y, t)$ and $\zeta_3(x, t)$ as follows:

$$\varphi_3(x, y, t) = -\frac{2g}{\pi} \int_0^{\infty} \frac{\cosh[k(y + H)]}{\omega(k) \cosh(kH)} \cos(kx)\, C(k) \int_0^t \dot{s}(\tau) \sin[\omega(k)(t - \tau)]\, d\tau\, dk,$$

$$\zeta_3(x, t) = -\frac{2}{\pi} \int_0^{\infty} \cos(kx) C(k)\omega(k) \int_0^t s(\tau) \sin[\omega(k)(t - \tau)]\, d\tau\, dk, \qquad (2.7)$$

$$C(k) = \frac{\tanh(kH)}{k}.$$

To determine the hydrodynamic pressure acting on the wall, we calculate the derivative of $(\partial\varphi/\partial t)\Big|_{x=0}$ using solutions (2.5)–(2.7):

$$\frac{\partial\varphi}{\partial t}\Big|_{x=0} = \frac{\partial\varphi_1}{\partial t}\Big|_{x=0} + \frac{\partial\varphi_2}{\partial t}\Big|_{x=0} + \frac{\partial\varphi_3}{\partial t}\Big|_{x=0}$$

$$= -\frac{2g}{\pi} \int_0^{\infty} \frac{\cosh[k(y + H)]}{\cosh(kH)} \tilde{\zeta}_0(k) \cos[\omega(k)t]\, dk - \frac{8H\ddot{s}(t)}{\pi^2} \sum_{n=0}^{\infty} (-1)^n \frac{\cos[\lambda_n(y + H)]}{(2n + 1)^2}$$

$$- \frac{2g}{\pi} \int_0^{\infty} \frac{\cosh[k(y + H)]}{\cosh(kH)} C(k) \int_0^t \dot{s}(\tau) \cos[\omega(k)(t - \tau)]\, d\tau\, dk. \qquad (2.8)$$

Integrating Eq. (2.8) with respect to y from $-H$ to 0, we can find the force acting on the vertical wall in Eq. (2.3). Finally, we obtain the integrodifferential equation for $s(t)$:

$$(m + m_a)\ddot{s} + \gamma s + \int_0^t \dot{s}(\tau) M(t - \tau)\, d\tau = Q(t). \qquad (2.9)$$

Here

$$M(t) = \frac{2g\rho}{\pi} \int_0^{\infty} \frac{\tanh^2(kH)}{k^2} \cos[\omega(k)t]\, dk,$$

$$Q(t) = \frac{2g\rho}{\pi} \int_0^{\infty} \frac{\tilde{\zeta}_0(k)}{k} \tanh(kH) \{1 - \cos[\omega(k)t]\}\, dk, \qquad m_a = \frac{16}{\pi^3} \rho H^2 \alpha, \qquad \alpha = \sum_{n=0}^{\infty} \frac{1}{(2n + 1)^3} = 1.05179974\ldots.$$

After determining the function $s(t)$, we can calculate the free surface elevation as

$$\zeta(x, t) = \frac{4s(t)}{\pi} \operatorname{arctanh}\Big[\exp\Big(-\frac{\pi x}{2H}\Big)\Big] + \frac{2}{\pi}\Big(\int_0^{\infty} \cos(kx)\, \tilde{\zeta}_0(k) \cos[\omega(k)t]\, dk$$

$$- \int_0^{\infty} \cos(kx)\, C(k)\omega(k) \int_0^t s(\tau) \sin[\omega(k)(t - \tau)]\, d\tau\, dk\Big).$$

It should be noted that the integrodifferential equation (2.9), which describes the vibrational motion of the wall, can also be obtained directly from the equation of body motion, which is used in the following form in the hydrodynamic theory of ship motion, starting from [5]:

$$(m + A^*)\ddot{s} + \gamma s + \int_0^t \dot{s}(\tau)N(t - \tau)\,d\tau = Q(t). \tag{2.10}$$

In this equation,

$$A^* = \lim_{\sigma\to\infty} A(\sigma), \qquad N(t) = \frac{2}{\pi}\int_0^\infty B(\sigma)\cos(\sigma t)\,d\sigma, \tag{2.11}$$

and the function $Q(t)$ is an external force. The functions $A(\sigma)$ and $B(\sigma)$ are called the added mass and damping coefficients. These functions were determined in [6] for the problem of wave generation by small vibrations of a vertical wall with a frequency σ and have the following form:

$$A(\sigma) = 2\rho\sum_{n=1}^{\infty}\frac{\sin^2(k_nH)}{k_n^2[k_nH + (1/2)\sin(2k_nH)]}, \qquad B(\sigma) = \frac{2\rho\sigma\sinh^2(k_0H)}{k_0^2[k_0H + (1/2)\sinh(2k_0H)]}.$$

Here k_0 and k_n $(n = 1, 2, \ldots)$ are the positive real roots of the equations $\sigma^2 = gk_0\tanh(k_0H)$ and $\sigma^2 = -gk_n\tan(k_nH)$, respectively.

It can be easily seen that

$$\lim_{\sigma\to\infty} k_n = \frac{\pi}{H}\Big(n - \frac{1}{2}\Big)$$

at high-frequency vibrations, and the limiting value of the added mass coefficient is $A^* = m_a$. Replacing the variable of integration for the function $N(t)$ in Eq. (2.11), we obtain $N(t) = M(t)$. Hence, Eqs. (2.9) and (2.10) are identical.

The analytical solution is compared below with numerical results obtained by CBEM in the linear and nonlinear cases.

3. Application of CBEM. This method used to solve steady problems of the flow past bottom obstacles by a finite-depth heavy fluid with a free surface was described in [7]. Solutions of some unsteady problems with free boundaries in a full nonlinear formulation can be found in [8, 9].

In the numerical solution of the problem considered, the flow domain $\Omega(t)$ in Eq. (1.2) is replaced by the computational domain $D(t)$ bounded by the free surface $C_1(t)$ and the solid walls: horizontal bottom C_2 $(y = -H)$, moving vertical wall $C_3(t)$ $[x = s(t)]$, and motionless vertical wall C_4 mounted at a sufficiently large distance from the wall $C_3(t)$ $(x = L)$. In the domain $D(t)$, it is required to determine the analytical function of the complex potential

$$w(z, t) = \varphi(x, y, t) + i\psi(x, y, t) \qquad (z = x + iy \in D), \tag{3.1}$$

where $\varphi(x, y, t)$ is the velocity potential and $\psi(x, y, t)$ is the stream function.

On the motionless solid boundaries C_2 and C_4, the unknown function $w(z, t)$ has to satisfy the condition of impermeability

$$\psi = 0 \qquad (z \in C_2,\, C_4);$$

on the free boundary $C_1(t)$, the function has to satisfy the kinematic and dynamic conditions

$$\frac{dz}{dt} = \frac{\partial\varphi}{\partial x} + i\,\frac{\partial\varphi}{\partial y}, \qquad \frac{\partial\varphi}{\partial t} + \frac{1}{2}\,|\nabla\varphi|^2 + gy = 0 \qquad (z \in C_1). \tag{3.2}$$

The following condition is imposed on the moving left wall C_3:

$$\psi(s(t), y, t) = (y + H)\dot{s}(t). \tag{3.3}$$

The algorithm of solving the problem in the nonlinear formulation includes the following stages. First, the stream function $\psi(x, y, t)$ in Eq. (3.1) on the solid walls C_2, C_3, and C_4 is specified. After that, the initial position of the free boundary C_1 and the zero distribution of the potential on this boundary are specified in accordance

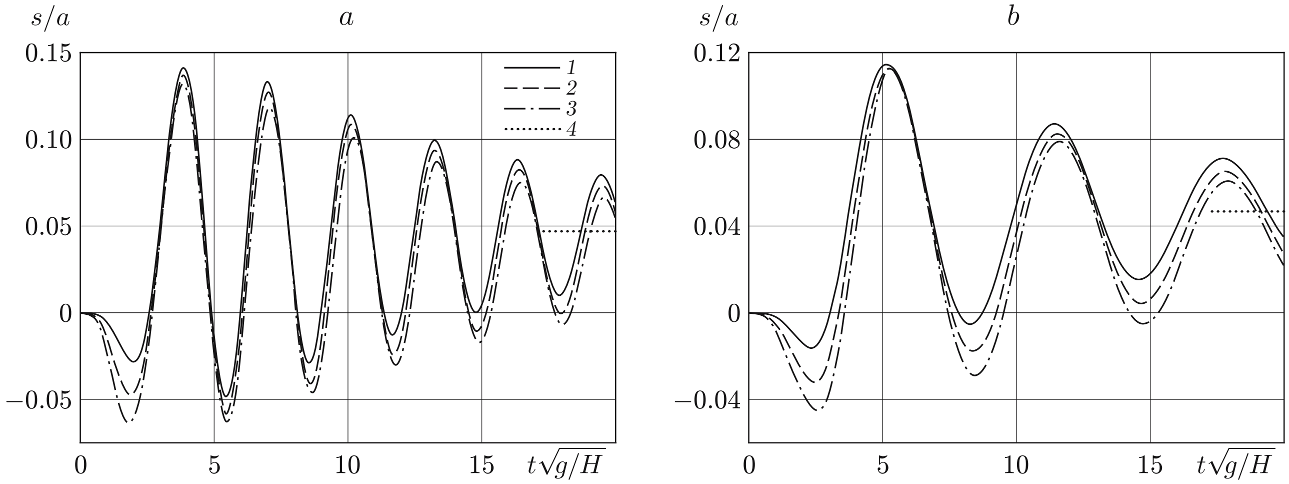

Fig. 2. Wall position versus time at $\bar{\gamma} = 5$ and $\bar{m} = 1$ (a) and 5 (b); the curves show the linear solution (1), the nonlinear solutions with $a/H = 0.1$ (2) and 0.2 (3), and the limiting values of the function $s(t)/a$ at $t \to \infty$ obtained in the linear approximation (4).

with the first condition in Eq. (1.9). The complex potential $w(z, t)$ over the entire boundary of the domain is found by applying CBEM. Then, the horizontal component $u = \partial\varphi/\partial x$ and vertical component $v = \partial\varphi/\partial y$ of the flow velocity on the domain boundary are computed, and $u = \dot{s}$ is assumed on the moving wall C_3. After that, the pressure is calculated using the known velocity field. For this purpose, it is necessary to solve an additional boundary-value problem with respect to the function $\partial w/\partial t$, which is analytical in the flow domain:

$$\frac{\partial w}{\partial t} = \frac{\partial \varphi}{\partial t} + i\,\frac{\partial \psi}{\partial t} \qquad (z \in D); \tag{3.4}$$

$$\frac{\partial \varphi}{\partial t} = -\frac{1}{2}\,(u^2 + v^2) - gy \quad (z \in C_1); \qquad \frac{\partial \psi}{\partial t} = 0 \quad (z \in C_2, C_4); \qquad \frac{\partial \psi}{\partial t} = v\dot{s} + (y + H)\ddot{s} \qquad (z \in C_3). \tag{3.5}$$

Using CBEM to calculate the analytical function (3.4) with the boundary conditions (3.5), we find the value of $\partial\varphi/\partial t$ on the boundary C_3. After that, the pressure is found from Eq. (1.8), and the force acting on the moving vertical wall is found from Eq. (1.11). By solving Eq. (1.10) by the Euler method, we calculate the wall displacement s, its velocity $\dot{s}$, and acceleration $\ddot{s}$. Using the boundary conditions (3.2), we calculate the new position of the free boundary and the distribution of the potential on this boundary. The moving left wall is shifted in accordance with the found value of s, and the stream function distribution on this wall is determined from condition (3.3). After that, this procedure is repeated at the next time step.

In solving the linear problem, the domain occupied by the fluid is assumed to be fixed, and the boundary C_1 coincides with the undisturbed free surface $y = 0$. The kinematic and dynamic conditions on the free surface (3.2) take the form (2.1) The calculation algorithm coincides with the algorithm of solving the nonlinear problem, with the only difference that the force acting on the left wall is determined from relation (2.3) rather than (1.11).

4. Results of Numerical Calculations. We performed calculations for the initial elevation of the free surface $\zeta_0(x)$ [see Eq. (1.1)] in the form of a localized elevation with an amplitude a:

$$\zeta_0(x) = \begin{cases} (a/2)[1 + \cos(\pi(x - x_0)/d)], & |x - x_0| \leqslant d, \\ 0, & |x - x_0| > d. \end{cases} \tag{4.1}$$

The Fourier transform of this function has the form

$$\tilde{\zeta}_0(k) = \frac{a\pi^2 \cos(kx_0)\sin(kd)}{k[\pi^2 - (dk)^2]}.$$

The results of numerical calculations presented below were obtained with the following parameters: $x_0/H = 0.7$ and $d/H = 0.5$.

It should be noted that the vertical wall in a fluid at rest and with a horizontal free surface is subject to hydrostatic pressure only. If there is an elevation on the free surface at the initial time, however, the hydrostatic

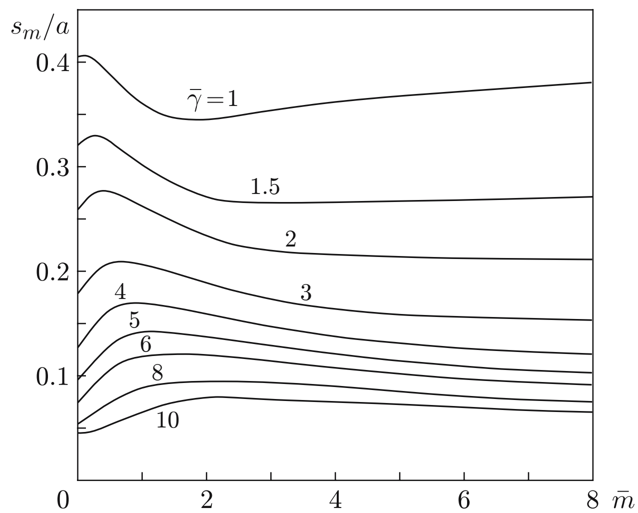

Fig. 3. Maximum deviations of the wall versus its mass, obtained in the linear approximation with different values of the spring stiffness coefficient.

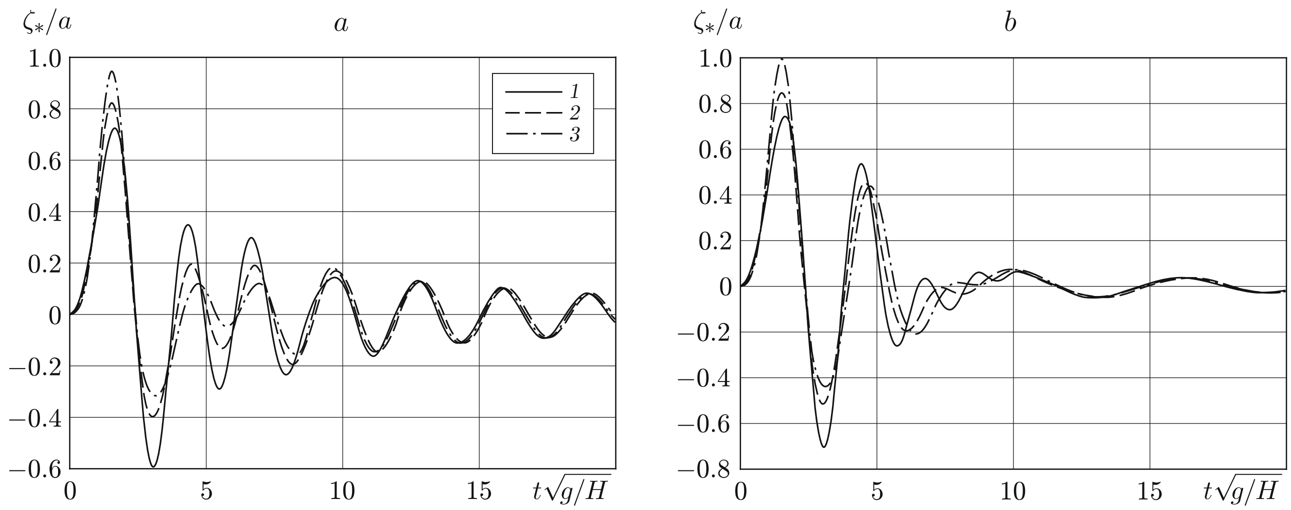

Fig. 4. Level of the fluid on the moving wall at $\bar{\gamma} = 5$ and $\bar{m} = 1$ (a) and 5 (b); the curves show the linear solution (1) and the nonlinear solution with $a/H = 0.1$ (2) and 0.2 (3).

pressure is supplemented by the term $\rho(\partial\varphi/\partial t)|_{x=0,t=0}$. This pressure is equal to zero at $y = \zeta_0(x)$, but differs from zero on the wall $x = 0$, $-H \leqslant y < 0$. With a chosen initial elevation (4.1), the wall is shifted to the left under the action of this additional pressure. This position is used as the zero value on the x axis. Wall vibrations decay with time, the free surface close to the wall returns to the undisturbed horizontal position, and the wall takes the position $x = s_0$ determined by the hydrostatic pressure with the initial elevation being ignored. In the linear case, this value is easily calculated from Eq. (2.8); for the chosen values of x_0 and d, this value is equal to $s_0 \approx 0.2334a\rho gH/\gamma$.

The integrodifferential equation (2.9) was solved numerically by the fourth-order Runge–Kutta method, and numerical integration was performed by the Simpson rule. In CBEM calculations, the length of the domain D was $L/H = 30$, the number of nodes on each vertical wall was 40, the number of nodes on the horizontal wall was 600, and the time step Δt was chosen to be constant: $\Delta t\sqrt{g/H} = 0.001$. In nonlinear calculations, the flow domain varied in time owing to the motion of the vertical wall, and the Lagrangian approach was used to describe the trajectories of motion of the free surface points.

Figure 2 shows the wall positions versus time at $\bar{\gamma} = \gamma/(\rho gH) = 5$ and $\bar{m} = m/(\rho H^2) = 1$ and 5, determined in both linear and nonlinear formulations. The linear solutions obtained by the complex boundary element method and by integrating Eq. (2.9) differ by less than 0.1%. The most significant difference between the linear and nonlinear solutions is observed at the initial times, when the wall moves to the left. When the wall moves to the right, there is a minor difference in the solutions. The dotted straight lines in Fig. 2 show the limiting values of the functions $s(t)/a$ at $t \to \infty$, which were obtained from the solution of the linear problem.

The maximum deviations of the wall to the right $s_m = \max s(t)$ as functions of its mass, which were obtained in the linear approximation for different values of $\bar{\gamma}$, are plotted in Fig. 3. For a fixed value of $\bar{\gamma}$, these dependences are nonmonotonic and have a local maximum at comparatively low values of $\bar{m}$; for a fixed value of $\bar{m}$, however, the value of s_m monotonically decreases with increasing $\bar{\gamma}$.

Figure 4 shows the time evolution of the free surface level on the wall ζ_*/a. It should be noted that substantial differences between the solutions, which are observed in the initial periods of vibrations, decrease with time, and the fluid flows predicted by the linear and nonlinear models coincide at large times.

Conclusions. Analytical and numerical solutions were obtained for a two-dimensional unsteady hydroelastic problem of interaction between surface waves and a moving vertical wall fixed on springs. The solution of the problem within the framework of the nonlinear model of potential flow was obtained by the complex boundary element method, and the linear problem was solved by two methods with the use of both integral transformations and CBEM. In the linear approximation, the results predicted by both methods coincide with each other. A comparison of the linear and nonlinear solutions shows that the linearized model can be used to predict some important characteristics of the flow under consideration.

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